International Journal of Mathematics
and Computer Applications Research (IJMCAR)
ISSN (P): 2249-6955; ISSN (E): 2249-8060
Vol. 8, Issue 5, Oct 2018, 1-12

A HOMOTOPY ANALYSIS METHOD TO FLOW AND HEAT TRANSFER CHARACTERESTICS OF A VISCO-ELASTIC FLUID THROUGH POROUS MEDIUM OVER AN EXPONENTIALLY STRETCHING SHEET WITH VISCOUS DISSIPATION

HYMAVATHI TALLA & B. AKKAYYA

Department of Mathematics, Adikavi Nannaya University, Rajamahendravaram, Andhra Pradesh, India

ABSTRACT

In this paper, a detailed analysis is carried out to study the flow and heat transfer characteristics of visco-elastic fluid over an exponentially stretching sheet in a porous medium with viscous dissipation. The governing partial differential equations are converted into non-linear ordinary differential equations using suitable similarity transformation and then non-linear differential equations have been solved by using Homotopy Analysis Method (HAM), which provides the best possible way to control and adjust the convergence region using a non-zero auxiliary parameter ħ. The effect of physical parameters on velocity and temperature is discussed through graphs explicitly.

***KEYWORDS:** Viscos-Elastic Fluid, Porous Medium, Exponentially Stretching Sheet, Viscous Dissipation & HAM*

Received: Jul 27, 2018**; Accepted:** Aug 17, 2018**; Published:** Aug 29, 2018**; Paper Id.:** IJMCAROCT20181

Original Article

INTRODUCTION

In recent years, the boundary layer flow of non – Newtonian fluids over a stretching sheet plays an important role in applications in industries and in many engineering problems such as metallurgical process, a polymer extrusion process involving cooling of a molten liquid being stretched into a cooling system. The pioneer work of Sakiadis [1] analysed the boundary layer flow over a continuously moving solid surfaces of infinite length and moving with constant speed. Crane [2] who extended the work of Sakiadis [1] on over stretching sheet such that the velocity of the surface to vary linearly with the distance from the slit. Later, many researchers laid their attention on the study of boundary layer flow of Newtonian and non – Newtonian fluids over different stretching sheets, i. e. linear, quadratic, power law and non-isothermal stretching sheet [3-6].

The study of heat transfer of non-Newtonian fluid flows through a porous medium has gained considerably in the industrial applications of petroleum extraction, filtration process and separation process in chemical industries and many others. Initially, Cheng. p and Minkowycz [7] analyzed free thermal convection from a vertical plate embedded in a saturated porous medium, considering wall temperature as a power-law function of the distance along the plate. Consequently, Subhash Abel and Veena [8] Examined heat transfer of a visco-elastic fluid flow through a porous medium adjacent to a stretching sheet including the effects of frictional heating and internal heat generation or absorption. Subsequently, diffusion of chemical reactive species of a non-Newtonian fluid was investigated by Prasad et al. [9] through the porous medium over a stretching sheet. Krishnambal and Anuradha [10] examined radiation effects on heat transfer of visco-elastic fluid flow through

porous medium past a stretching sheet. M. Subhas Abel, S. K. Khan and K. V. Prasad [11] studied combined buoyancy effects to flow, heat and mass transfer in a visco-elastic fluid through a porous medium over a stretching sheet.

The problem of viscous dissipation, Joule heating and heat source/sink on non-Darcy MHD natural convection flow over a flux permeable sphere in a porous medium is numerically analyzed by Yih [12]. The work of Sonth et al. [13] analyzed the effect of the viscous dissipation term along with temperature dependent heat source/sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface. Later, Chen [14] examined the effect of combined heat and mass transfer on MHD free convection from a vertical surface byohmic heating and viscous dissipation. Recently, Mahentesh M. Nandeppanavar [15] considered the flow and heat transfer characteristics of a visco – elastic fluid through a porous medium over an impermeable stretching sheet with viscous dissipation.

The solutions given for stretching sheet in the above literature were not unique and derived another closed form of solutions. Consequently, Troy et al.[16] investigated momentum flow of visco-elastic fluid containing exponential terms in boundary conditions. The results obtained were physically realistic produces a boundary layer only slightly altered in its dimensions from the viscous one. In view of the above discussions on boundary conditions, we present in the next section the physically realistic sequential similarity solutions of visco-elastic boundary layer. In view of this point, Elbashbeshy[19] analysed the problem of heat transfer over an exponentially stretching sheet with suction confined only to viscous fluid. Subsequently, Sanjayanand and Khan [19, 20] extended the work of Elbashbeshy [18] to visco - elastic fluid flow, heat and mass transfer over an exponentially stretching sheet.

The investigation of exact solutions to the problem of nonlinear equations plays an important role in the study of the nonlinear physical phenomena. Firstly, the Homotopy Analysis Method [21, 22] was proposed by Liao in 1992. Further, the HAM was developed and improved by Liao for nonlinear problems in [23]. In this process, Sajid and Hayet [24] discussed the influence of thermal radiation on the boundary layer flow due to an exponential stretching sheet. Recently, Hymavathi. T [25] et. al analysed the flow and heat transfer in visco-elastic fluid through a porous medium over an exponentially stretching sheet. They used Homotopy Analysis Method (HAM) to solve the problem analytically.

In view of the importance of a porous medium, we investigate to study of the heat transfer of visco-elastic fluid flow over exponentially stretching sheet in porous medium with viscous dissipation. Here we employed Homotopy Analysis Method to solve converted non-linear differential equations and discuss the effect of visco – elastic parameter (k_1), porosity parameter (k_2), Prandtl number (Pr) and Eckert number (Ec) on the velocity and temperature profiles. Comparison of the present analysis is also made with the existing results in the literature and is seen in good agreement.

MATHEMATICAL FORMULATION

We consider steady state two-dimensional incompressible visco-elastic fluid over exponential stretching sheet through porous medium with viscous dissipation. The flow will be generated by applying two equal and opposite force along x-axis such that the origin is fixed and the flow confined to y>0. The sheet is stretched with speed varying exponentially with a distance from the slit. We take x-axis is along the surface and y- axis being perpendicular to it. The governing equations for the flow are defined as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\vartheta}{k^*}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left[\left(\frac{u^2}{k^*}\right)+\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{3}$$

Where ϑ is the kinematic viscosity, k_0 is the elastic parameter and k^* is the permeability of porous medium. Where k is the thermal conductivity, ρ is the density, T is the temperature, Cp is the specific heat at constant pressure and $\mu\left[\left(\frac{u^2}{k^*}\right)+\left(\frac{\partial u}{\partial y}\right)^2\right]$ is viscous dissipation.

The boundary conditions for the velocity fields are of the form:

$u = U_w(x) = U_0\, e^{x/l},\ v = 0,\ T = T_w = T_\infty + T_0\, e^{x/l}$ at $y = 0$

$u = 0,\ u_y = 0,\ T = T_\infty,$ as $y \rightarrow \infty$ (4)

where T_w is the temperature of the sheet, T_∞ is the temperature of the fluid far away from the sheet and l is the characteristic length.

SOLUTION OF THE PROBLEM

The velocity components u and v in terms of stream function $\psi(x, y)$ can be written as:

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{5}$$

For solving momentum equation, introduce a similarity variable η such that

$$\eta\ =\ y\sqrt{\frac{U_0}{2\nu l}}e^{x/2l} \tag{6}$$

$$\psi(x,y)=\sqrt{2\nu l U_0}\, f(x,\eta)e^{x/2l},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{7}$$

Here f is the dimensionless stream function and considering *if* $(x, \eta) = f(\eta)$. Using (5) to (7), the equations (2) & (3) results in a fourth order non- linear ordinary differential equation of the form

$$2f'^2-f'f''=f'''-k_1\left[3f'f'''-\frac{1}{2}ff''''-\frac{3}{2}f''^2\right]-k_2 f' \tag{8}$$

and

$$\theta''-\Pr(2f'\theta-f\theta')+k_2\Pr Ecf'^2+\Pr Ecf''^2=0 \tag{9}$$

Where $k_1 = \frac{k_0 U_w}{vl}$, $k_2 = \frac{2vl}{k^* U_w}$, $\Pr = \frac{\mu c_p}{k}$, $Ec = \frac{U_0 u_w}{A c_p}$ are the visco-elastic parameter, porosity parameter, Pr and t l number and Eckert number respectively.

Subject to the boundary conditions are

$f = 0$, $f' = 1$, $\theta = 1$, at $\eta = 0$

$$f' = f'' = \theta = 0, \text{ as } \eta \to \infty \tag{10}$$

ANALYTICAL SOLUTION (HAM)

In this section, we employ HAM to solve the equations (8) to (9) subject to the boundary conditions (10). We choose the initial guesses f_0, θ_0 of f, θ in the following form:

$$f_0(\eta) = 1 - e^{-\eta} \tag{11}$$

$$\theta_0(\eta) = e^{-\eta} \tag{12}$$

The linear operators are selected as

$$L_1(f) = f''' - f' \tag{13}$$

$$L_2(\theta) = \theta'' - \theta \tag{14}$$

which have the following properties

$$L_1\left(C_1 + C_2 e^{-\eta} + C_3 e^{\eta}\right) = 0 \tag{15}$$

$$L_2\left(C_4 e^{-\eta} + C_5 e^{\eta}\right) = 0 \tag{16}$$

Where C_i *(i = 1 to 5)* are arbitrary constants. If $q \in [0,1]$ is the embedding parameter, $\hbar_1$ *and* $\hbar_2$ are the non – zero auxiliary parameters and $H_1(\eta)$, $H_2(\eta)$ are the auxiliary functions, then we can construct the zeroth – order deformation equations

$$(1-q)L_1\left(f(\eta,q) - f_0(\eta)\right) = q\hbar_1 H_1(\eta) N_1\left(f(\eta,q)\right) \tag{17}$$

$$(1-q)L_2\left(\theta(\eta,q) - \theta_0(\eta)\right) = q\hbar_2 H_2(\eta) N_2\left(f(\eta,q), \theta(\eta,q)\right) \tag{18}$$

subject to the boundary conditions

$$\begin{aligned} &f(0,q) = 1, \quad f'(0,q) = 1, \quad f'(\infty,q) = 0 \\ &\theta(0,q) = 1, \quad \theta'(\infty,q) = 0 \end{aligned} \tag{19}$$

Where

$$N_1\left(f(\eta,q)\right)=\frac{\partial^3 f}{\partial\eta^3}-2\left(\frac{\partial f}{\partial\eta}\right)^2+\frac{\partial^2 f}{\partial\eta^2}f-k_1\left(3\frac{\partial f}{\partial\eta}\frac{\partial^3 f}{\partial\eta^3}-\frac{1}{2}f\frac{\partial^4 f}{\partial\eta^4}-\frac{3}{2}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2\right)-k_2\frac{\partial f}{\partial\eta} \tag{20}$$

$$N_2\left(f(\eta,q),\theta(\eta,q)\right)=\frac{\partial^2\theta}{\partial\eta^2}+\Pr\left(f\frac{\partial\theta}{\partial\eta}\right)-2\Pr\frac{\partial f}{\partial\eta}\theta+Ec\Pr\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2+Ec\Pr k_2\left(\frac{\partial f}{\partial\eta}\right)^2 \tag{21}$$

$$\begin{aligned} &f(\eta,0)=f_0(\eta), && f(\eta,1)=f(\eta)\\ &\theta(\eta,0)=\theta_0(\eta) && \theta(\eta,1)=\theta(\eta) \end{aligned} \tag{22}$$

Thus, as q increases from 0 to 1, $f(\eta,q)$ varies from $f(\eta,0)$ to $f(\eta)$ and $\theta(\eta,q)$ varies from $\theta(\eta,0)$ to $\theta(\eta)$. Then expanding $f(\eta,q),\theta(\eta,q)$ using Taylor's theorem with respect to q, we obtain

$$f(\eta;q)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)q^m \tag{23}$$

$$\theta(\eta;q)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)q^m \tag{24}$$

Where

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta;q)}{\partial q^m}\Big|_{q=0} \tag{25}$$

$$\theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\hat{\theta}(\eta;q)}{\partial q^m}\Big|_{q=0} \tag{26}$$

The auxiliary parameters are properly chosen so that series (23) and (24) converges at $q=1$ and thus

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta) \tag{27}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta) \tag{28}$$

The resulting problems at the mth order deformation are

$$L_1\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=h_f R_m^f(\eta), \tag{29}$$

$$L_2\left[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)\right]=h_\theta R_m^\theta(\eta) \tag{30}$$

Subject to the boundary conditions

$$f_m(0)=0,\ f'_m(0)=0,\ f'_m(\infty)=0,\theta_m(0)=0,\theta_m(\infty)=0 \tag{31}$$

$$R_m^f = f'''_{m-1} + \sum_{k=0}^{m-1} f_k f''_{m-1-k} - 2\sum_{k=0}^{m-1} f_k' f'_{m-1-k} - k_1\left(3\sum_{k=0}^{m-1} f'_{m-1-k} f_k''' - \frac{1}{2}\sum_{k=0}^{m-1} f_{m-1-k} f_k'''' - \frac{3}{2}\sum_{k=0}^{m-1} f''_{m-1-k} f_k''\right) - k_2 f'_{m-1}. \tag{32}$$

$$R_m^\theta(\eta) = \theta''_{m-1} + Pr\sum_{i=0}^{m-1} f_{m-1-i}\,\theta_i' + Ec\,\mathrm{Pr}\, f''_{m-1} + Ec\,\mathrm{Pr}\, k_2\, {f'_{m-1}}^2 \tag{33}$$

and

$$\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1 \end{cases} \tag{34}$$

We choose the auxiliary function as follows:

$$H_1(\eta)=1\ , H_2(\eta)=1. \tag{35}$$

If we let $f_m^*(\eta)\,,\theta_m^*(\eta)$ as the special solutions of the m[th] order deformation equations, the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + \mathrm{C}_1 + \mathrm{C}_2\mathrm{e}^{-\eta} + \mathrm{C}_3\mathrm{e}^{\eta} \tag{36}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + \mathrm{C}_4\mathrm{e}^{-\eta} + \mathrm{C}_5\mathrm{e}^{\eta} \tag{37}$$

Where the integral constants $C_i(i=1to5)$ are determined using the boundary conditions (31).

The linear non-homogenous equations (29) and (30) are solved using MATHEMATICA software one after the other by considering m = 1,2,.............

CONVERGENCE OF HAM SOLUTION

Liao [16] showed that for an analytic solution obtained by HAM, its convergence and rate approximation strongly depending upon the auxiliary parameters $\hbar_1$, and $\hbar_2$. If these parameters are chosen properly, then the solution is effective. Hence, $\hbar$-curves are plotted at 25[th] order approximation in order to obtain the suitable ranges for $-1.15 \le \hbar_1 \le -0.1$, $-1.1 \le \hbar_2 \le -0.09$, with convergence of h value is h = -0.8. Table 1 shows the convergence of the solutions with increasing order of approximations.

RESULTS AND DISCUSSIONS

A comprehensive parametric study is considered for flow and heat transfer of visco-elastic fluid through porous medium with viscous dissipation over exponentially stretching sheet. The approximate analytical solutions are obtained by the Homotopy Analysis Method for physical parameters like visco-elastic parameter, porous parameter, Pr and tl number and Eckert number. A systematic study is performed to analyze the effect of various values for physical parameters visco-elastic, porosity, pr and tl and Eckert number on velocity and temperature through graphically. We now discuss the variations of the physical quantities of engineering importance that is the local skin friction coefficient *–f "(0)* and local Nusselt number *–θ'(0)*. The convergence values of *-f''(0)* and *–θ'(0)* displayed in Table 1.

Table 1: Convergence of HAM Solution for different Orders of Approximations when

$$k_1 = 0.1, k_2 = 0.1,\ Ec = 0.1, \Pr = 1.0$$

Order	$-f''(0)$	$-\theta'(0)$
5	1.424645	1.203512
10	1.427125	1.200031
15	1.427156	1.199619
20	1.427156	1.199521
25	1.427156	1.199492
30	1.427156	1.199483
35	1.427156	1.199481
40	1.427156	1.199481

Table 2: Comparison of –θ'(0) for different Values of k_1 with k_2 = 0.5, Pr = 1.0, Ec = 0.25

K_1	Mahentesh M	Present Study
0.0	1.12221	1.122209
0.1	1.09631	1.0963096
0.2	1.06551	1.065513

From the figure 1 shows the admissible values of *f'' (0)* and *θ' (0)* at 25th order approximation on velocity and temperature profiles.

From the figures 2 and 3, shows the effect of visco-elastic parameter on velocity and temperature. It can be found that the increase in visco-elastic parameter decreases the velocity as well as increase in temperature. This is due to the fact that elastic property in visco-elastic fluid reduces the frictional force.

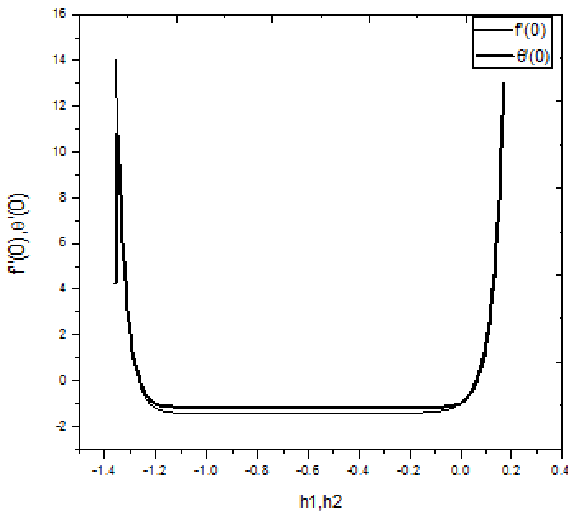

Figure 1: 25th Order Approximations for h1, h2 Curves

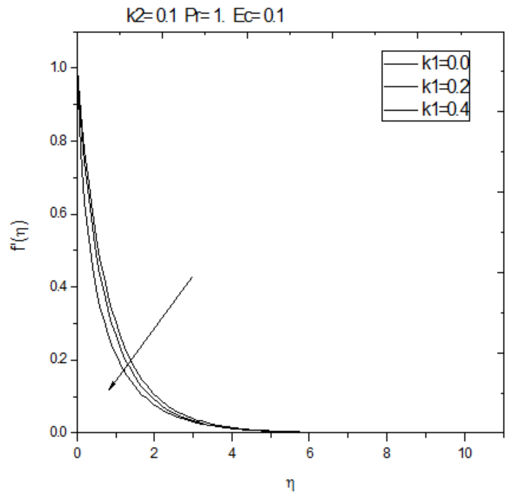

Figure 2: The Effect of Visco-Elastic Parameter on Velocity

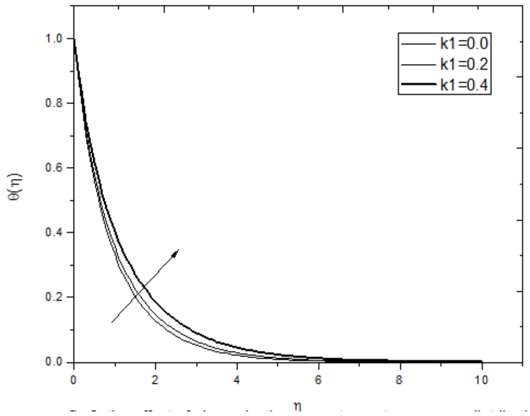

Figure 3: The Effect of Visco-Elastic Parameter on Temperature Distribution

From figures 4 and 5, shows the effect of porosity parameter on velocity and temperature distribution. It can be found that the increase in porosity parameter decreases the velocity as well as an increase in temperature. This is due to increase in porosity parameter leads to the enhanced deceleration of the flow, thickening of the thermal boundary layer occurs due to the increase of non-Newtonian visco-elastic normal stress and the effect of dimensionless porous parameter becomes smaller as porous parameter increases.

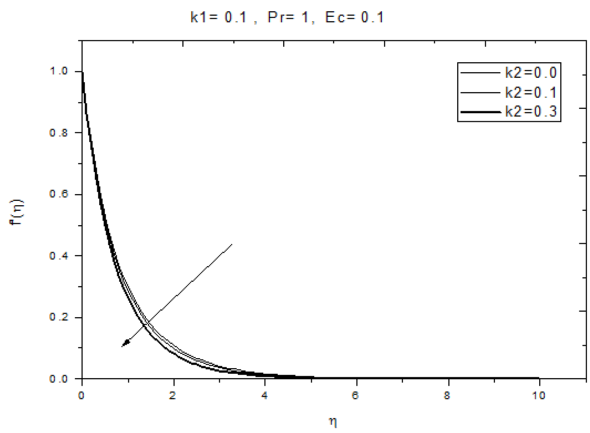

Figure 4: The Effect of Porosity Parameter on Velocity

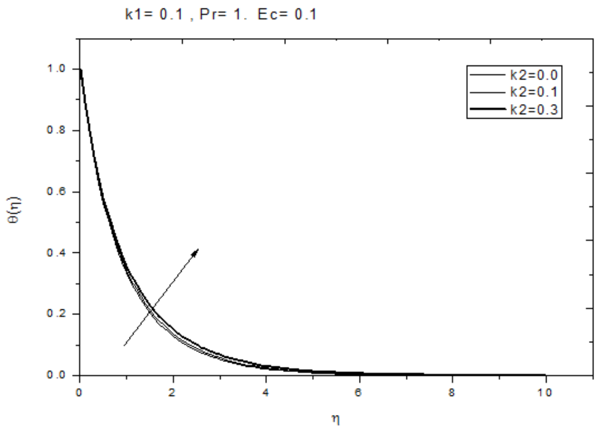

Figure 5: The Effect of Porosity Parameter on Temperature

From the figures 6 and 7, shows the effect of Pr and tl number and Eckert number on temperature distribution.

It can be found that increase in Pr and tl number decreases the temperature distribution and increases in

Eckert number increases the temperature distribution. This is because there would be a decrease of the thermal boundary layer thickness with the increase of values of Pr and tl number Pr. The increase of Pr and tl number means slow rate of thermal diffusion.

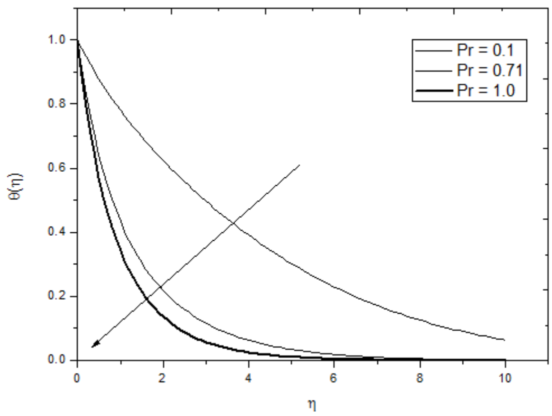

Figure 6: The Effect of Pr and t1 Number on Temperature Distribution

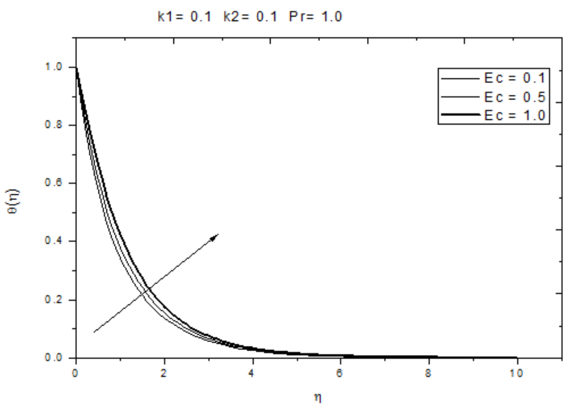

Figure 7: The Effect of Eckert Number on Temperature

CONCLUSIONS

In this study, Homotopy Analysis Method is employed to analyse the flow and heat transfer characteristic of visco-elastic fluid over exponentially stretching sheet in a porous medium with viscous dissipation. Analytical solutions obtained by the Homotopy Analysis method (HAM) are seen in good agreement with existing results in the literature. The effect of various parameters on velocity and temperature is analyzed through graphs. The results are summarized as follows.

- The effect of visco-elastic and porosity parameter on horizontal velocity are the same.
- The effect of Pr and tl number is to decrease the thermal boundary layer thickness.
- The effect of Eckert number increases the temperature distribution
- The visco-elastic liquids having low viscous dissipation must be chosen for effective cooling of stretching sheet.

REFERENCES

1. *Sakiadis, B. C., Boundary Layer Behaviour on Continuous Solid Surface: I – Boundary Layer Equations for Two Dimensional and Axisymmetric Flows, AIChE. J., 7 (1961), 1, pp. 26-28.*
2. *L. J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21 (1970) 645–647.*
3. *V. Kumaran, G. Ramanaiah, A note on the flow over a stretching sheet, Acta Mech. 116 (1996)229–233.*
4. *K. Vajravelu, J. R. Cannon, Fluid flow over a nonlinear stretching sheet, Appl. Math. Comput. 181 (2006) 609–618.*
5. *H. I. Andersson, MHD flow of a viscoelastic fluid past a stretching surface, Acta Mech. 95 (1992) 227–230.*
6. *T. Hymavathi, B. Sankar, Heat transfer in MHD visco-elastic fluid flow over non-isothermal stretching sheet, Journal of Appl. Mathematics and Computation 215(2009)2045-2054.*
7. *Cheng P, Minkowycz WJ (1977) Free convection about a vertical plate embedded in a porous medium with application to heat transfer from a dick. J. Geo. phys. Res 28:2040–2048*
8. *Subhas, A., Veena, P., Viscoelastic Fluid Flow and Heat Transfer in a Porous Medium over a Stretching Sheet, Int. J. Nonlinear Mech. 33 (1998), 3, pp. 531-540.*
9. *Prasad, K. V., Abel, M. S., and Datti, P. S. (2003). Diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet, Int. J. Non- Linear Mech., 38, 651–657.*
10. *Krishnambal, S., and Anuradha, P. (2006). Effect of radiation on the flow of a visco-elastic fluid and heat transfer in a porous medium over a stretching sheet, J. Appl. Sci., 6(14), 2901–2906.*
11. *M. Subhas, Abel, S. K. Khan and K. v. Prasad, Convective heat and mass transfer in a visco- elasticfluid flow through a porous medium over a stretching sheet, Int. Journal of Numerical Methods for Heat & Fluid flow, Vol.11 No.8,2001, pp.779-792.*
12. *K. A. Yih, Viscous and Joule heating effects on Non-Darcy MHD natural convection flow overa permeable sphere in porous media with internal heat generation, Int. Commun. Heat Mass,**27(4)**, pp. 591–600, 2000.*
13. *R. M. Sonth, S. K. Khan, M. S. Abel, K. V. Prasad, Heat and mass transfer in a visco-elastic fluid over an accelerating surface with heat source/sink and viscous dissipation, Heat Mass Transfer, **38**, pp. 213–220, 2002.*

14. C. H. Chen, Combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation, Int. J. Eng. Sci., **42**, pp. 699–713, 2004.

15. Mahantesh M. Nandeppanavar, M. Subhas Abel, K. Vajravelu, Flow and heat transfer characterestics of a visco- elastic fluid in a porous medium over an impermeable stretching Sheet with viscous dissipation, Int. J. of Heat and Mass transfer 53(2010)7407-4713.

16. W. C. Troy, E. A. Overmanii and G. B. Ermentrout, Uniqueness of flow of a second order fluid past a stretching sheet, Quarterly of Applied Mathematics, Volume XLIV, Number4, January 1987.

17. Elbashbeshy, E. M. A., Heat Transfer Over an Exponentially Stretching Continuous Surface with Suction, Arch. Mech., 53 (2001), 6, pp. 643-651.

18. Rajalaxmi, V., Mohankumar, G., & Ramanathan, K. Effectiveness Of Plantar Fascia Stretching Vs Contrast Bath Combined With Ultrasound In Plantar Fasciitis.

19. Khan, S. K., Sanjayanand, E., Viscoelastic Boundary Layer Flow and Heat Transfer over an Exponentially Stretching Sheet, Int. J. Heat Mass Transfer, 48 (2005), 8, pp. 1534-1542.

20. Sanjayanand, E., Khan, S. K., On Heat and Mass Transfer in a Viscoelastic Boundary Layer Flow over an Exponentially Stretching Sheet, Int. J. Therm. Sci., 45 (2006), 8, pp. 819-828.

21. Liao SJ. The proposed homotopy analysis technique for the solution of nonlinear problems (Ph. D. thesis), Shanghai Jiao Tong University; 1992.

22. S. J. Liao, A new branch of solutions of boundary-layer flows over an impermeable stretched plate, International Journal of Heat & Mass Transfer 48(2005), 2529 - 2539.

23. Liao SJ. Beyond perturbation: introduction to the homotopy analysis method. Boca Raton: Chapman and Hall/CRC Press; 2003.

24. Wu W, Liao SJ. Solving solitary waves with discontinuity by means of the homotopy analysis method. Chaos Solitons Fractals 2005;26:177–85.

25. Sajid, M., Hayat, T., Influence of Thermal Radiation on the Boundary Layer Flow Due to an Exponentially Stretching Sheet, Int. Comm. Heat Mass Transfer, 35 (2008), 3, pp. 347-356.

26. T. Hymavathi and B. Akkayya, Homotopy Analysis method to flow and heat transfer flow in visco-elastic fluid flow in porous medium over exponentially stretching sheet, Int. Journal of Computational and Appl. Mathematics, Vol.11No.2(2016), pp.119-128.